Impact of Q-matrix Misspecification on Cognitive Attributes Estimation: A Simulation Study of Comparing Three Estimation Approaches

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Research Questions

1. What is the performance of the cross-random effect linear logistic test model (CRE-LLTM), compared with the linear logistic test model (LLTM) and the two-stage multiple regression method in terms of the accuracy and precision of parameter estimation for cognitive attributes?
2. What is the impact of Q-matrix misspecification on the performance of three approaches under the over-, under-, and balanced-misspecification conditions?

Methodology

In this simulation study the data were generated based on the crossed-random effects LLTM using the SAS/IML software. The population distributions included a normal distribution (skewness = 0, kurtosis = 0), a negatively-skewed distribution (skewness = -1, Kurtosis = 3) and a positively-skewed distribution (skewness = 1, kurtosis =3). The manipulated sample sizes were 100, 250, and 1000. For each condition, 1000 replications were generated. Two LLTM-type approaches were fitted to the simulated data using the SAS GLIMMIX procedure. The procedures for two-stage multiple regression were first to obtain Rasch item difficulty and then regressed them on cognitive attributes as predictors.

The sparse and dense Q-matrices used for this study were extracted from Fischer and Formman (1972) and Medina-Diaz (1993), respectively, totaling 21 items with 8 cognitive attributes. The sparse Q-matrix had only 34 out of 160 entries that contained 1s (approximately 21.25%). In contrast, the dense Q-matrix had 92 out of 160 entries as 1s (approximately 57.5%). Each Q-matrix had its own set of true attribute parameters. The percentages of misspecification in the Q-matrix were 2%, 6%, and 10%. Three types of misspecification were over-misspecification (0s→1s), under-misspecification (1s→0s), and balanced-misspecification (0s→1s and 1s→0s). Three criteria were used to evaluate their estimation performance, including bias, root mean square error (RMSE), and correlation between the estimated and true sets of parameters. Finally, factorial ANOVA analyses with generalized eta squares (cutoff value =.058) were conducted to examine what manipulated variables affect bias, RMSE, and correlations.

Results

The simulation results indicate that the CRE-LLTM model with the SAS GLIMMIX procedure performs very well evidenced by small bias and RMSE and high correlations between the estimated and true sets of parameters when the Q-matrix is specified correctly across various conditions. Under conditions of Q-matrix misspecification, CRE-LLTM provided more accurate and precise estimate of cognitive attribute than LLTM and two-stage multiple regression, with smaller bias and RMSE and higher correlations. Bias and RMSE of Over- and under-misspecification were higher than those of the balanced-misspecification. The higher the percentage of misspecification of the Q-matrix, the less accurate and precise the estimate of the attributes were.